Symmetry of Microwave Devices with Gyrotropic Media-Complete Solution and Applications

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***Abstract*—In this paper, a general procedure for constructing all the possible solutions for symmetrical devices and components with gyrotropic medias is suggested. Using the theory of symmetry and crystallographic principles, all the color groups and corresponding matrices $[S]$, $[Z]$, and $[Y]$ of the devices can be obtained. With this approach, it is possible to select those symmetrical structures and magnetic fields, which can be considered as candidates in the process of synthesis of microwave devices and components. In order to illustrate the procedure, some examples are given.**

I. INTRODUCTION

MANY types of ferrite and magnetized semiconductor devices in the microwave region have been suggested during the last 40 years. Different physical effects are utilized in such devices. Among these effects may be named ferromagnetic resonance, Faraday rotation, field-displacement, nonreciprocal phase-shift, Cotton–Mouton effect, and others.

Along with rigorous solutions of boundary problems some engineering approaches, for example the theory of microwave circuits, are widely used for investigation of devices with gyrotropic medias. Among the existing methods, the symmetry theory occupies a special place. This theory supplies exact information because no approximations are made under its application. The symmetry principles are often utilized to verify the validity of new physical laws. The theory of symmetry can provide a good deal of general and useful information by means of simple calculations. Sometimes this information is unique and cannot be obtained by other methods.

The theory of electromagnetic field symmetry in gyrotropic media has been considered by many authors [1]–[3]. Several papers of the author [4]–[7] have been devoted to the application of crystallography methods and principles to microwave circuits with gyrotropic media. It has been shown that color groups, the Curie's and Newmann's principles known in crystallography, may be successfully used in investigation of microwave circuits and for the synthesis of devices.

Color (magnetic) groups include the symmetry in space–time coordinates, and the time coordinate is connected with the presence of an external dc magnetic field. Curie's principle defines the symmetry of a complex structure. In this case, it leads to a superposition of the geometry symmetry and symmetry of the dc magnetic field. Newmann's principle in crystallography deals with the connection of a property tensor symmetry and the geometrical symmetry of a crystal. The matrices $[S]$, $[Z]$, and $[Y]$ may be considered as tensors and connect the space–time symmetry of a device with the symmetry of its parameter matrices. Hence, there is an analogy here: tensor–matrix, crystal–device. A theoretical basis of the application of Newmann's principle to microwave circuits is given in [7].

Using these approaches and also the concept of gyrotropic symmetry (GS) and gyrotropic antisymmetry (GA) suggested in [4], it is possible to reduce the number of independent elements of parameter matrices ($[S]$, $[Z]$, and $[Y]$) appreciably, to predict some of the properties of devices, to explain effects in novel devices, and to check the correctness of solving a problem by other methods and the results of computations and measurements. It allows one to make a first step in the synthesis of devices.

It will be shown in this paper how to find all the possible solutions for symmetrical gyrotropic structures using these principles, i.e., all the possible parameter matrices. The problem of determination of the structure of the dc magnetic field will be also discussed. Several examples will be given in order to illustrate the theory. One of them is a quadratic waveguide, where polarization effects are possible. Some properties of such waveguides will also be investigated in this paper.

This approach may be used for any symmetrical structure independently on the type of waveguide or transmission line and physical effect being used.

II. A GENERAL APPROACH TO THE PROBLEM

The problem of searching such a structure which is capable of fulfilling a certain microwave function (or several ones simultaneously) may be formulated in different ways. It may be given the ideal matrix of a device (a component) only and it then becomes necessary to find geometrical structures and dc magnetic fields which allow one to get the best approximation to this matrix. This problem has been considered in [7]. The structure of the dc magnetic field may be fixed and the task is to find a geometry of the device. At last, the geometrical structure may be given and it is necessary to find the symmetry of the dc magnetic field in order to get the desired matrix. This situation exists, for example, in waveguides and transmission lines. Here, the last problem will be considered, though the results of this paper may be used for solving all the mentioned problems.

Usually when symmetry of a scattering matrix is discussed it means its symmetry about the main diagonal. In this paper, the symmetry of $[S]$ shall be considered with a broad meaning,

Manuscript received March 15, 1996; revised November 21, 1996.

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Publisher Item Identifier S 0018-9480(97)01716-X.

namely in space–time coordinates. The symmetry of a matrix about the main diagonal is the symmetry of time only. But if $[S]$ is treated as a tensor, it is possible to consider symmetry of the matrix under a different coordinate transformation, (i.e., if there is such symmetry, the matrix is not changed under certain coordinate transformations). In a general case, symmetry about both changing space coordinates and time needs to be considered.

On designing different microwave elements with gyrotropic media the following procedure may be suggested:

1) determine the nonmagnetic group of symmetry of the given geometrical structure;
2) using Curie's principle, define the nonmagnetic group of symmetry of the system: geometrical structure plus isotropic ferrite element, if these symmetries are different;
3) define all the possible magnetic groups of the second and the third category using the diagram in Fig. 8;
4) determine generators of the chosen magnetic groups using Table II;
5) calculate the matrices $[S]$ for all magnetic groups using GS and GA commutation relations;
6) choose those groups of symmetry which provide us with the necessary matrix;
7) define the structure of the dc magnetic field.

Further investigations may include the choice of a necessary physical effect, calculations using the unitary condition, electrodynamic calculations, and so on. In the following sections, the main steps of the above algorithm will be considered.

III. A Description of Magnetic Groups of Symmetry

To begin an investigation of symmetrical devices with gyrotropic medias, systematic information about magnetic point groups is needed, which shall be applied first to crystallographic point groups. There are three categories of such magnetic groups [8].

1) The 32 groups including the operator of the time inversal T. They describe nonmagnetic crystals. The matrix $[S]$ in this case is symmetrical about the main diagonal.
2) The 32 groups without any form of the time reversal T. It is a case of magnetic media, the matrix $[S]$ in general is nonsymmetric about the main diagonal, but for finding the relations between some matrix elements, there are only GS cases, and the commutation relations (identities) coincide with those for nonmagnetic media.
3) The 58 groups (real magnetic groups) which include the operator T only in combination with spacial operations. These groups also describe magnetic media. There exist two types of identities, namely, for the cases of GS and GA [5], which can be used for the calculation of $[S]$.

In this paper, the concern is with the second and the third categories of the magnetic groups. To treat these groups, the table of crystallographic groups in two notations is shown in Table II [8]; additionally, the number of elements for every group is also given. The Schubnikov and Schoenflies notations used in Table II complement each other. The Schoenflies notation shows the whole group and its subgroups of elements with and without the operation of time inversion. The Schubnikov's notation is very useful for the purposes of this paper because it indicates generating elements of symmetry (i.e., axes, antiaxes, planes, and antiplanes). For instance, in the group $D_{4h}(D_{2d})\,(m\cdot\underline{4}:\underline{m})$ there are four-fold antiaxis $\underline{4}$, a plane of symmetry m which coincides with the antiaxis 4, and an antiplane of symmetry $\underline{m}$ which is perpendicular to the antiaxis. The generating matrices may be chosen corresponding to these three elements. Hence, by using Table II one can find necessary generating matrices.

In order to find all the possible solutions, information about subgroup decomposition is also needed. The diagram of decomposition of the 32-point crystalligraphic groups is not widely available, so it shall be quoted in Fig. 8. In comparison with the diagram given in [8] dotted lines have been added here which correspond to the subgroups of index $n \neq 2$.

The crystallographic magnetic groups which have been described above do not comprise all of the possible symmetries which can be met in practice. For instance, the magnetic groups C_5, C_7, and others are not presented in Table II and on the diagram. Generally speaking, there are an infinite number of point groups. The whole system of the nonmagnetic point groups of symmetry is given in [9] including the cases of an infinite number of elements. To find magnetic noncrystallographic point groups, a known algorithm [10] may be used.

IV. Symmetry and Structure of the DC Magnetic Field

Notice that the problem of determination of a symmetry group of the given dc magnetic field $\mathbf{H}_0$ (or the current producing this field) is unique but the inverse problem is not.

If the magnetic group of symmetry is known, how may a corresponding structure of $\mathbf{H}_0$ be found? An infinite number of solutions may in fact be found for the problem for every given magnetic group (not violating, of course, the Gauss' law). For the group $C_s(\underline{m})$ for instance, the dc magnetic field may be oriented parallel to the antiplane of symmetry in any direction, or normal to the antiplane but in opposite directions on both sides of it, and so on. Another example: how to get only one plane of symmetry $z = 0$? One of the possible solutions is as follows. The direction of $\mathbf{H}_0$ in this case may be perpendicular to the plane $z = 0$ and $\mathbf{H}_0$ must vary nonsymmetrically in this plane.

Sometimes, it is more convenient to solve the problem using the symmetry of electric current which produces the desired dc magnetic field. In this case, it is necessary to bear in mind that electric current is a polar vector whereas $\mathbf{H}_0$ is an axial vector and in mapping there must be a distinction between them.

In some cases, it is useful to consider structures of the dc magnetic field for the continuous magnetic groups. If a group of symmetry contains an ∞-fold axis, it is a continuous group. This group is indicated in the Schoenflies' notation by C_∞ and by ∞ in the Schubnikov's notation. There are seven continuous magnetic groups of the second category and seven continuous magnetic groups of the third category [11]. In order to give an interpretation to these groups, one can imagine them as some geometrical models. Several examples

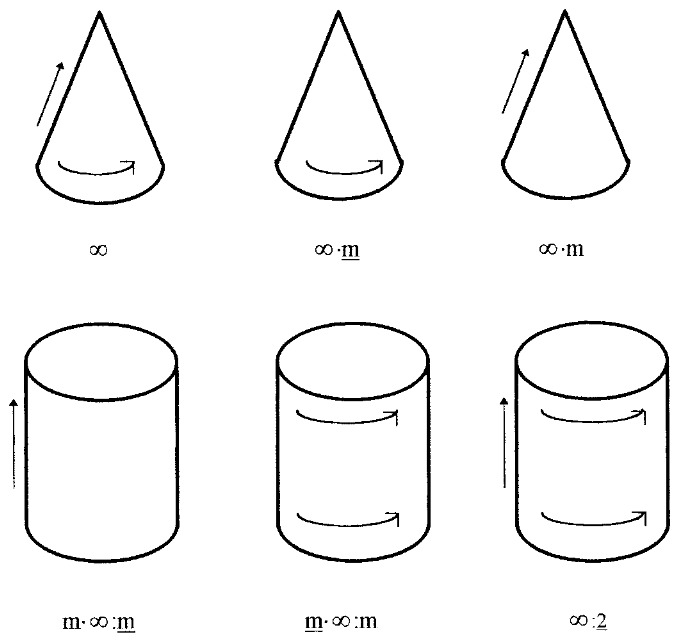


Fig. 1. Geometrical models for continuous magnetic groups of the second and the third category.

of such models are shown in Fig. 1. The arrows on the models denote the directions of currents which produce a dc magnetic field. The magnetic field with the symmetry $m \cdot \infty : \underline{m}$ for example, may be produced by a current line, the field with the symmetry $\underline{m} \cdot \infty : m$—by a coil with direct current, the field with the symmetry $\infty : \underline{2}$—by applying these two sources simultaneously, etc.

Solving the problem of finding the necessary structure of the dc magnetic field with that continuous magnetic group for which the given group of geometrical symmetry of the device is a subgroup may now begin. In the process of solving the problem the continuous group may be lowered in order to obtain the necessary matrix $[S]$.

To construct the structure of the dc magnetic field, the following method may be useful. A minimal symmetrical volume (a part of the device) of the given structure may be defined and the necessary structure of $\mathbf{H}_0$ set at this time. In other symmetrical parts of the device, the field $\mathbf{H}'_0$ is calculated as follows. For the GS:

$$\mathbf{H}'_0(\mathbf{r}') = (\det[\gamma])\,[\gamma]\,\mathbf{H}_0(\mathbf{r})$$

and for GA:

$$\mathbf{H}'_0(\mathbf{r}') = -(\det[\gamma])\,[\gamma]\,\mathbf{H}_0(\mathbf{r})$$

where $[\gamma]$ are the matrices which define symmetrical transformations [3].

An analogy with the work [12] may be seen here where a minimum sector of waveguide cross section is defined for solving the whole problem. But in this problem, the symmetry of the electromagnetic field may be discussed only in the case of GS and nothing may be said about the symmetry of the electromagnetic field in the case of GA. Reference in the last case can only be made about the antisymmetry of the dc magnetic field $\mathbf{H}_0$.

Notice that the number of sources of the dc magnetic field may be different. It may be one common source for all symmetrical parts of the device or there may be several of them for different parts of the device. This problem needs special consideration in every specific case. It particularly depends on the physical effect being used.

V. Illustration I—Ferrite Waveguides with Different Symmetries

The types of waveguides and transmission lines are many and diversified. Over 100 different types of these key elements of any microwave system have been suggested using different physical effects, on the basis of many of them nonreciprocal and control ferrite elements and components are designed. They are isolators, filters, phase shifters, nonreciprocal quarter-wave plates, electronic rotable half-wave plates, etc.

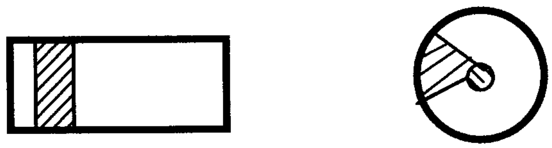
Using the symmetry theory in [12] some properties of uniform waveguides are discussed. In this paper, only the symmetry of the cross section of waveguides is taken into consideration and the length of the waveguide is infinite. Hence, the problem is two-dimensional. The point groups which describe objects with all three finite dimensions shall be used here, thus dealing with a three-dimensional (3–D) problem.

To clarify the approach discussed so far, some specific examples shall be considered beginning with uniform waveguides with gyromagnetic medias. There are only four types of possible matrices $[S]$ in the cases of waveguides without polarization effects [14]:

$$\begin{vmatrix} S_{11} & S_{12} \\ S_{12} & S_{11} \end{vmatrix} \quad \begin{vmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{vmatrix}$$
$$\begin{vmatrix} S_{11} & S_{12} \\ S_{21} & S_{11} \end{vmatrix} \quad \begin{vmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{vmatrix}.$$

The cross sections of waveguides may have the symmetries C_n and C_{nv} ($n = 0, 1, 2, \cdots, \infty$). When $n = \infty$, the limit case which describes a circular waveguide is present. A section of the waveguides with some dielectric, magnetodielectric, or metal insertions may have different symmetries, including continuous groups C_∞, D_∞, $C_{\infty h}$, $C_{\infty v}$, and $D_{\infty h}$ (the groups D_{nh} and D_{nd} with $n \to \infty$ transform themselves into one group $D_{\infty h}$).

Proceeding from the highest symmetry G of the given waveguide section with isotropic media in Fig. 8 downwards, all the possible magnetic groups for this waveguide may be found. To define all the groups of the third category, starting with a certain group G, there must be a descent from G until the group C_1 along all the possible lines disregarding dotted and heavy lines. These lines correspond to the subgroups of index $n \neq 2$ and to noninvariant subgroups accordingly. For the groups of the second categories, all the possible lines including dotted and heavy ones must be gone through. The whole number of groups of the second category is equal to the number of groups which are presented in the corresponding tree. The number of groups of the third category is equal to the number of thin lines of the tree. Thus, all the possible



C_1 - symmetry of the cross-section,

C_s - symmetry of the waveguide section.

Possible magnetic groups of the second category:

C_s, C_1.

Possible magnetic groups of the third category:

$C_s(C_1)$.

The group tree:

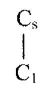
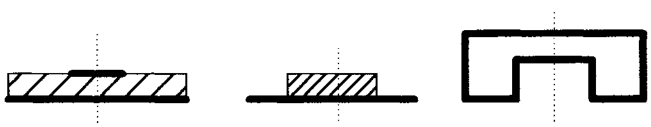


Fig. 2. Examples of the waveguides with the symmetry C_s. (a) Rectangular waveguide. (b) Coaxial line. The group tree and possible magnetic groups of the second and the third category for these waveguides.



C_{1v} - symmetry of the cross-section,

C_{2v} - symmetry of the waveguide section.

Possible magnetic groups of the second category:

C_{2v}, C_2, C_s, C_1.

Possible magnetic groups of the third category:

$C_{2v}(C_2)$, $C_{2v}(C_s)$, $C_2(C_1)$, $C_s(C_1)$.

The group tree:

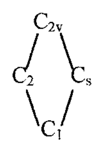
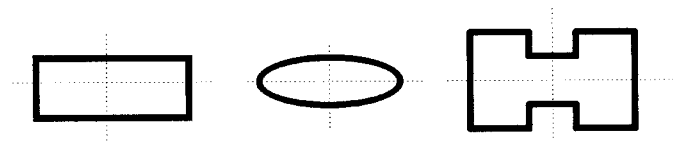


Fig. 3. Examples of the waveguides with the symmetry C_{2v}. (a) Microstrip line. (b) Image line. (c) Single-ridged waveguide. The group tree and possible magnetic groups of the second and the third category for these waveguides.

solutions lie in the symmetry tree between the group G on the top and the group C_1 at the bottom.

Several specific symmetries of existing waveguides shall now be examined. In Figs. 2–4 different types of waveguides, their symmetries and possible magnetic groups of the second and the third categories, are presented. The cross sections of the waveguides in Fig. 2 have the symmetry C_1. A section of such isotropic waveguides may have only one plane of symmetry which is perpendicular to the longitudinal axis of the waveguides. Hence, this section can exhibit only the symmetry C_s. Starting from the group C_s in Fig. 8, only allows to go down to the group C_1. Hence, the group tree consists of the elements C_s and C_1. Possible magnetic groups of the second categories are C_s and C_1 and of the third category are $C_s(C_1)$. This completes the list of magnetic groups which are possible in the waveguides being examined.

Sections of the isotropic waveguides in Fig. 3 have the symmetry C_{2v}. The corresponding group tree consists of four



C_{2v} - symmetry of the cross-section,

D_{2h} - symmetry of the waveguide section.

Possible magnetic groups of the second category :

D_{2h}, D_2, C_{2h}, C_{2v}, C_i, C_2, C_s, C_1.

Possible magnetic groups of the third category:

$D_{2h}(D_2)$, $C_{2h}(C_{2h})$, $D_{2h}(C_{2v})$,
$D_2(C_2)$, $C_{2h}(C_i)$, $C_{2h}(C_2)$, $C_{2h}(C_s)$, $C_{2v}(C_2)$, $C_{2v}(C_s)$,
$C_i(C_1)$, $C_2(C_1)$, $C_s(C_1)$.

The group tree:

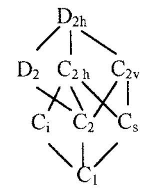


Fig. 4. Examples of the waveguides with the symmetry D_{2h}. (a) Rectangular metal or dielectric waveguide. (b) Elliptical dielectric transmission line. (c) Double-ridged waveguide. The group tree and possible magnetic groups of the second and the third category for these waveguides.

elements: C_{2v}, C_2, C_s, and C_1. All the possible magnetic groups for this case are given in Fig. 3.

Several waveguides with higher symmetry D_{2h} are shown in Fig. 4. In this case, the group tree is thicker and there are eight groups of the second category and 12 groups of the third category.

From Table II, generators of the groups may be found and then using corresponding commutation relations calculate the matrices $[S]$, $[Z]$, and $[Y]$ [5].

It is an easy matter to construct tables (peculiar catalogues) of all possible symmetries and to obtain corresponding parameter matrices for all existing waveguides.

VI. Illustration II—Quadratic Ferrite Waveguide with Polarization Effects

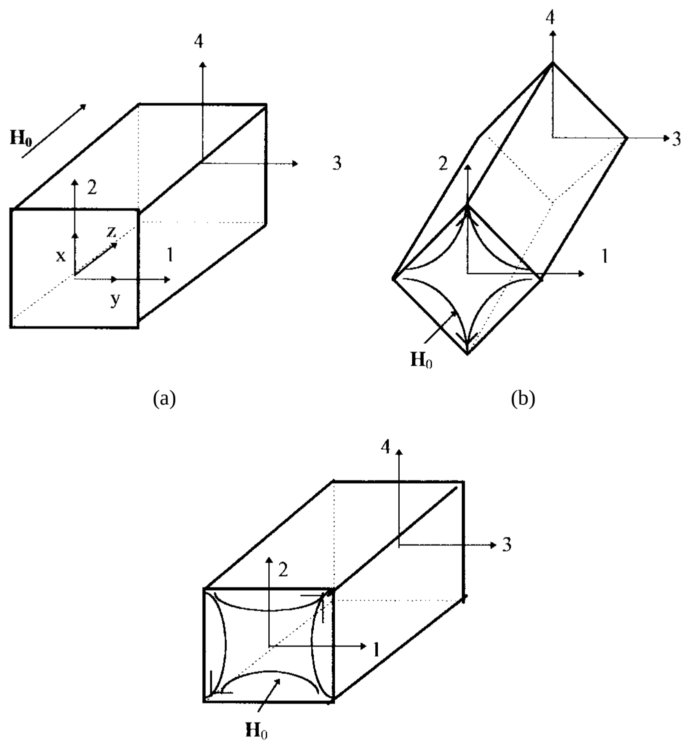
Attention should now be directed to the next specific example: to find those symmetries which can provide theoretically polarization effects in a quadratic gyrotropic waveguide (Fig. 5).

Two cases of such waveguides are well known in the literature: the waveguide with longitudinal dc magnetic field where the Faraday effect is observed and the waveguide with transverse quadrupole magnetic field where the Cotton–Mouton birefringence exists [6].

In order for a polarization effect to be possible in the waveguide, the matrix $[S]$ must have the following structure:

$$[S] = \begin{vmatrix} \bullet & \bullet & X & X \\ \bullet & \bullet & X & X \\ X & X & \bullet & \bullet \\ X & X & \bullet & \bullet \end{vmatrix}$$

where the crosses X signify that these matrix elements must be different from zero, and the dots signify that these elements are not important for the purpose of this discussion (on this stage of investigations).



$$[S]=\begin{vmatrix} s_{11} & 0 & s_{13} & 0 \\ 0 & s_{11} & 0 & s_{24} \\ s_{24} & 0 & s_{11} & 0 \\ 0 & s_{13} & 0 & s_{11} \end{vmatrix}$$

(c)

Fig. 5. Quadratic ferrite waveguides: (a) with longitudinal dc magnetic field, Faraday effect exists. The corresponding matrix $[S]_F$ is written in Table I, (b) with transverse quadrupole dc magnetic field, Cotton–Mouton effect exists. The corresponding matrix [S]C is written in Table I, and (c) with transverse quadrupole dc magnetic field, Cotton–Mouton effect is impossible.

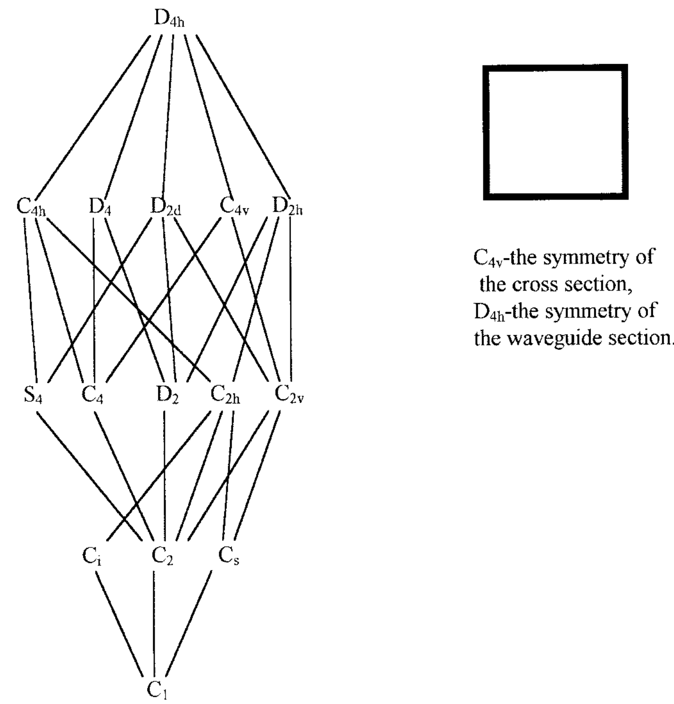


Fig. 6. The group tree for the group D_{4h} of the quadratic waveguide.

The symmetry of the cross section of the isotropic quadratic waveguide is C_4, the symmetry of its section is D_{4h}. In the Shubnikov's notation, this symmetry is $m \cdot 4 : m$. Hence, using three corresponding generating matrices, the following scattering matrix of the waveguide section may be found:

$$[S]=\begin{vmatrix} S_{11} & 0 & S_{13} & 0 \\ 0 & S_{11} & 0 & S_{13} \\ S_{13} & 0 & S_{11} & 0 \\ 0 & S_{13} & 0 & S_{11} \end{vmatrix}. \qquad (1)$$

This matrix has the simplest structure and contains two independent parameters. Between this simplest matrix and the most general matrix for the group E with 16 independent parameters, a variety of $[S]$ with a different structure and a different number of parameters lies.

Presented here is a reproduction of part of Fig. 8 (see Fig. 6), which corresponds to the decomposition of the group D_{4h}. It can be seen in this figure that in order to find the complete solution of the problem, 29 magnetic groups of the second category and 15 magnetic groups of the third category must be considered. For some of the groups [for example $D_{2h}(D_2)$, $D_{2h}(C_2)$, $D_{2h}(C_{2v})$] and for groups with lower symmetries several variants of symmetry elements orientations must be considered. For example, in the group $D_2(C_2)\,(2:\underline{2})$, it is necessary to investigate three cases: when the axis 2 is oriented along the x, y, or z coordinate axes and in each of these variants there are two subvariants with two different orientation of the antiaxis $\underline{2}$ (some of these variants and subvariants, however, may turn out to be equivalent). Three variants of different orientation of antiaxis 2 exist in the group $C_2\,(C_1)$ and three variants of orientation of the antiplane $\underline{m}$ in the group $C_s\,(C_1)$. The four-fold axes 4, $\underline{4}$, $\overline{4}$, and $\underline{\overline{4}}$ may be oriented only along the waveguide axis z and the choice of x and y axes is arbitrary. Therefore, for those groups where these axes are present, only one variant is possible.

For every magnetic group there must be calculated (for a fixed $\mathbf{H}_0$) two possible orientations of the ports [Fig. 5(b) and (c)] because the matrices $[S]$ in these two cases may be different. This situation exists, for example, in the waveguide with a quadrupole dc magnetic field. It may be seen from the corresponding matrix in Fig. 5(c) that the ports 1, 4 and 2, 3 are decoupled and a polarization effect is impossible. With such orientation of the ports, the normal modes of the waveguide are taken into consideration [13]. For the ports oriented as shown in Fig. 5(b), the polarization effect is possible because of the description in terms of orthogonal coupled modes. For the waveguide with Faraday effect, both of the ports' orientations lead to the identical matrices.

With the given symmetry D_{4h} of a section of the waveguide, the resultant symmetry by modifying the symmetry of the ferrite element and/or symmetry of $\mathbf{H}_0$ may be changed.

TABLE I
THE SCATTERING MATRICES FOR QUADRATIC FERRITE WAVEGUIDES DESCRIBED BY THE FIVE HIGHEST MAGNETIC GROUPS OF THE THIRD CATEGORY

Magnetic group of the third category	$D_{4h}(C_{4h})$ $\underline{m}\cdot 4:m$	$D_{4h}(D_4)$ $\underline{m}\cdot 4:\underline{m}$	$D_{4h}(D_{2d})$ $m\cdot\underline{4}:\underline{m}$	$D_{4h}(C_{4v})$ $m\cdot 4:\underline{m}$	$D_{4h}(D_{2h})$ $m\cdot\underline{4}:m$
Matrix [S]	$\begin{vmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ -S_{12} & S_{11} & -S_{14} & S_{13} \\ S_{13} & S_{14} & S_{11} & S_{12} \\ -S_{14} & S_{13} & -S_{12} & S_{11} \end{vmatrix}$	$\begin{vmatrix} S_{11} & S_{12} & S_{13} & 0 \\ -S_{12} & S_{11} & 0 & S_{13} \\ S_{13} & 0 & S_{11} & -S_{12} \\ 0 & S_{13} & S_{12} & S_{11} \end{vmatrix}$	$\begin{vmatrix} S_{11} & 0 & S_{13} & S_{14} \\ 0 & S_{11} & S_{14} & S_{13} \\ S_{13} & -S_{14} & S_{11} & 0 \\ -S_{14} & S_{13} & 0 & S_{11} \end{vmatrix}$	$\begin{vmatrix} S_{11} & 0 & S_{13} & S_{14} \\ 0 & S_{11} & 0 & S_{13} \\ S_{31} & 0 & S_{11} & 0 \\ 0 & S_{31} & 0 & S_{11} \end{vmatrix}$	$\begin{vmatrix} S_{11} & 0 & S_{13} & 0 \\ 0 & S_{11} & 0 & S_{13} \\ S_{13} & 0 & S_{11} & 0 \\ 0 & S_{13} & 0 & S_{11} \end{vmatrix}$
Number of independent parameters of [S]	4	3	3	3	2
Is polarization effect possible?	yes, Faraday's effect	no	yes, Cotton-Mouton effect	no	no, the matrix coincides with (1)

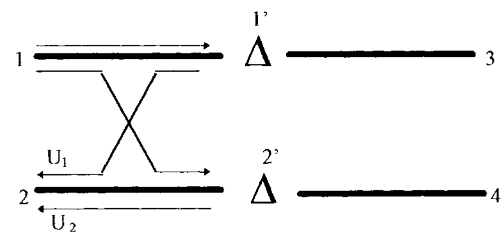


Fig. 7. An equivalent model of the waveguides with polarization effects.

The recording of all possible solutions of the problem requires excessive space. Presented here are only some of them which possess high symmetry. These cases are the most interesting. Notice that in order to find the matrix $[S]$, it is not necessary at this stage of investigation to know the real geometrical structure of the device and the structure of the dc magnetic field.

The results of the calculations for the five highest magnetic groups of the third category are given in Table I. Polarization effects (Faraday and Cotton–Mouton) are possible in two cases, Notice that in many cases of lower symmetries the existence of the polarization effects is expected as well, first of all in those groups which are subgroups of the group $D_{4h}(C_{4h})$ and $D_{4h}(D_{2d})$. It may also be assumed that in some cases of lower symmetries both of these effects exist.

VII. DISCUSSION

The above matrices may be analyzed by applying unitary conditions, but some results can be obtained under investigations of the matrices' structure. While comparing and scrutinizing the matrices $[S]_F$ and $[S]_C$ [Fig. 5(a) and (b)] each of these four-ports as two coupled lines may be considered: the first line connects the ports 1 and 3 and the second line connects ports 2 and 4 (Fig. 7).

The symmetry of 3–D finite objects may include some nonuniformity in the waveguides which does not change the symmetry group of the section. For example, the insertion of a circular section of a dielectric, ferrite, or metal rod on the axis of a waveguide with symmetry C_{nh} does not change the initial symmetry of the empty waveguide, but such a section will cause a reflection (which is inevitable in a frequency band). That is why the reflection coefficients S_{ii} in all matrices do not equal zero.

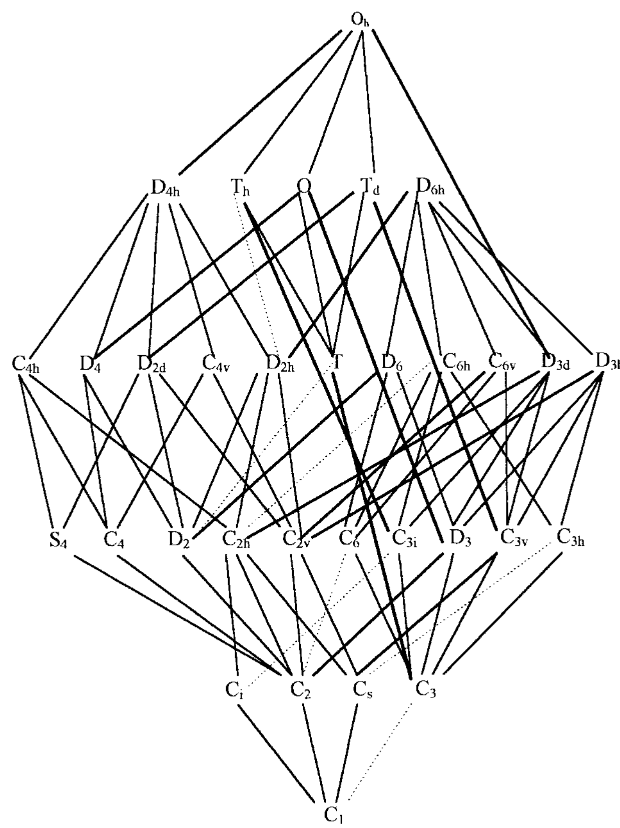


Fig. 8. Subgroup decomposition of the thirty-two point crystallographic groups. A heavy line indicates that the subgroup is not invariant, a dotted line indicates the subgroup is not of index 2 under the above group.

Notice first of all that the reflection coefficients S_{ii} in the lines 1–3 and 2–4 of the four-ports under consideration are equal. For the Faraday waveguide (F-waveguide), this result is obvious without the use of symmetry theory, but for the Cotton–Mouton waveguide (C-waveguide), it is not.

TABLE II
CRISTALLOGRAPHIC MAGNETIC POINT GROUPS

Shubnikov	Schoenflies	Number of elements
$\underline{\bar{2}}$	$C_i\,(C_1)$	2
$\underline{2}$	$C_2\,(C_1)$	2
$\underline{m}$	$C_s\,(C_1)$	2
$\underline{2}:\underline{m}$	$C_{2h}\,(C_i)$	4
$2:\underline{m}$	$C_{2h}\,(C_2)$	4
$\underline{2}:m$	$C_{2h}\,(C_s)$	4
$2\cdot\underline{m}$	$C_{2v}\,(C_2)$	4
$\underline{2}\cdot m$	$C_{2v}\,(C_s)$	4
$2:\underline{2}$	$D_2\,(C_2)$	4
$\underline{m}\cdot 2:m$	$D_{2h}\,(C_{2h})$	8
$\underline{m}\cdot 2:\underline{m}$	$D_{2h}\,(D_2)$	8
$m\cdot 2:\underline{m}$	$D_{2h}\,(C_{2v})$	8
$\underline{4}$	$C_4\,(C_2)$	4
$\underline{\bar{4}}$	$S_4\,(C_2)$	4
$\underline{4}:m$	$C_{4h}\,(C_{2h})$	8
$4:\underline{m}$	$C_{4h}\,(C_4)$	8
$\underline{4}:\underline{m}$	$C_{4h}\,(S_4)$	8
$4:\underline{2}$	$D_4\,(C_4)$	8
$\underline{4}:2$	$D_4\,(D_2)$	8
$4\cdot\underline{m}$	$C_{4v}\,(C_4)$	8
$\underline{4}\cdot m$	$C_{4v}\,(C_{2v})$	8
$\bar{4}\cdot\underline{m}$	$D_{2d}\,(S_4)$	8
$\underline{\bar{4}}\cdot\underline{m}$	$D_{2d}\,(D_2)$	8
$\underline{\bar{4}}\cdot m$	$D_{2d}\,(C_{2v})$	8
$\underline{m}\cdot 4:m$	$D_{4h}\,(C_{4h})$	16
$m\cdot\underline{4}:m$	$D_{4h}\,(D_{2h})$	16
$\underline{m}\cdot 4:\underline{m}$	$D_{4h}\,(D_4)$	16
$m\cdot 4:\underline{m}$	$D_{4h}\,(C_{4v})$	16
$m\cdot\underline{4}:\underline{m}$	$D_{4h}\,(D_{2d})$	16
$3:\underline{2}$	$D_3\,(C_3)$	6
$3\cdot\underline{m}$	$C_{3v}\,(C_3)$	6
$3:\underline{m}$	$C_{3h}\,(C_3)$	6
$\underline{\bar{6}}$	$C_{3i}\,(C_3)$	6
$\bar{6}\cdot\underline{m}$	$D_{3d}\,(C_{3i})$	12
$\underline{\bar{6}}\cdot\underline{m}$	$D_{3d}\,(D_3)$	12
$\underline{\bar{6}}\cdot m$	$D_{3d}\,(C_{3v})$	12
$\underline{m}\cdot 3:m$	$D_{3h}\,(C_{3h})$	12
$\underline{m}\cdot 3:\underline{m}$	$D_{3h}\,(D_3)$	12
$m\cdot 3:\underline{m}$	$D_{3h}\,(C_{3v})$	12
$\underline{6}$	$C_6\,(C_3)$	6
$6:\underline{2}$	$D_6\,(C_6)$	12
$\underline{6}:2$	$D_6\,(D_3)$	12
$6\cdot\underline{m}$	$C_{6v}\,(C_6)$	12
$\underline{6}\cdot m$	$C_{6v}\,(C_{3v})$	12
$\underline{6}:\underline{m}$	$C_{6h}\,(C_{3i})$	12
$6:\underline{m}$	$C_{6h}\,(C_6)$	12
$\underline{6}:m$	$C_{6h}\,(C_{3h})$	12
$\underline{m}\cdot 6:m$	$D_{6h}\,(C_{6h})$	24
$m\cdot\underline{6}:\underline{m}$	$D_{6h}\,(D_{3d})$	24
$\underline{\bar{6}}/2$	$T_h\,(T)$	24
$\underline{m}\cdot 6:\underline{m}$	$D_{6h}\,(D_6)$	24
$m\cdot 6:\underline{m}$	$D_{6h}\,(C_{6v})$	24
$m\cdot\underline{6}:m$	$D_{6h}\,(D_{3h})$	24
$3/\underline{\bar{4}}$	Td (T)	24
$3/\underline{4}$	O(T)	24
$\bar{6}/\underline{4}$	$O_h\,(T_h)$	48
$\underline{\bar{6}}/4$	$O_h\,(O)$	48
$\underline{\bar{6}}/\underline{4}$	$O_h\,(T_d)$	48

On the whole, both of the four-ports are nonreciprocal but the connections between the ports 1–3 and 2–4 are reciprocal. The pairs of ports 1–3 and 2–4 for both waveguides lie in the antiplanes of symmetry. In accordance with general theory [5], it leads to a reciprocal connection, and it is a sufficient condition for reciprocity.

In the matrix $[S]_F$, the relations $S_{32} = S_{14}$ and $S_{41} = S_{23}$ exist, but for the matrix $[S]_C$ the relations $S_{23} = S_{14}$ and $S_{41} = S_{32}$ are valid. Formally, this is explained by a different symmetry, i.e., by the different conditions under rotation by $\pi/2$ about z-axis and reflection in the plane $z = 0$. For the F-waveguide they are the GS-cases (the elements 4 and m), and for the C-waveguide GA cases (the elements $\underline{4}$ and $\underline{m}$). Physically, different effects are exhibited in these waveguides.

The connections between the ports 1 and 4, and also 2 and 3 are nonreciprocal, and the elements S_{14} and S_{41}, as well as S_{23} and S_{32} differ by the π-phase shift in both waveguides.

Another difference exists in the matrices $[S]_C$ and $[S]_F$. In the C-waveguide the ports 1 and 2, and also 3 and 4 are completely decoupled (i.e., $S_{12} = S_{21} = S_{34} = S_{43} = 0$) whereas for the F-waveguide these ports are coupled. A natural question appears: Why is it so? A possible explanation is as follows. Under availability of reflections and polarizations effects, a signal entering port 1 will cause a signal in port 2.

The signal from port 1 to port 2 goes by two paths $1 \rightarrow 1' \rightarrow 2$ and $1 \rightarrow 2' \rightarrow 2$. These two paths are shown in Fig. 7. The reflection coefficients at the points $1'$ and $2'$ are equal because of the symmetry. Hence, for the C-waveguide there are two reflected signals U_1 and U_2 in port 2:

$$\begin{aligned} U_1 &= kS_{11}S_{31}S_{23} \\ &= kS_{11}S_{13}S_{14} \\ U_2 &= kS_{11}S_{41}S_{24} \\ &= -kS_{11}S_{13}S_{14} \end{aligned}$$

where k is a constant. These signals are out of phase and their sum is equal to zero:

$$U_1 + U_2 = 0.$$

Therefore, the elements $S_{12} = S_{21} = S_{34} = S_{43} = 0$ and it is valid on every frequency.

For the F-waveguide an analogous consideration leads to the following relations:

$$\begin{aligned} U_1 &= -kS_{11}S_{13}S_{14} \\ U_2 &= -kS_{11}S_{13}S_{14} \\ U_1 + U_2 &= -2kS_{11}S_{13}S_{14} \end{aligned}$$

i.e., the sum of the signals U_1 and U_2 does not equal zero. If $S_{11} = 0$, the elements S_{12}, S_{21}, S_{34}, S_{43} obviously must also be equal to zero. Under the condition $S_{11} = 0$, both of the devices have the matrix of a nonreciprocal directional coupler.

This consideration shows that from the viewpoint of matching, the C-waveguide is more preferable.

As it has been said in [5] the nonreciprocity which appears as a result of the analysis is not a sufficient condition. The symmetry analysis leaves the relation between elements S_{ik} and S_{ki} uncertain and in general this connection may be nonreciprocal but it may be reciprocal depending on the mutual orientation $\mathbf{H}_0$ and alternating magnetic field $\mathbf{h}$. In particular, the nonreciprocity and existence of polarization effects may depend on the type of mode which propagates in the waveguide. If this approach gives the negative result (i.e., impossibility of solving the problem), it is the final diagnoses. When the result is promising, further investigations are necessary to prove the feasibility of the device.

VIII. Conclusion

In this paper, all the results of the parameter matrices have been obtained as a consequence of the geometrical symmetry and the symmetry of the dc magnetic field. These results are not related to a specific physical effect or a specific type of waveguide.

The problems of theoretical feasibility of devices and the explanation of the principles of their functioning can sometimes be made clear by using the symmetry theory. Symmetry analysis also allows parameter matrices to be obtained and to find out some of the general properties of components and devices. It helps to select those structures which can be suitable for the fulfillment of different microwave functions and thereby to solve the first stage of the synthesis problem. This can be readily accomplished by means of simple calculations.

Acknowledgment

The author is pleased to thank Dr. L. A. S. Romo Saltos, Director of the Center of Scientific Research for supporting this work.

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